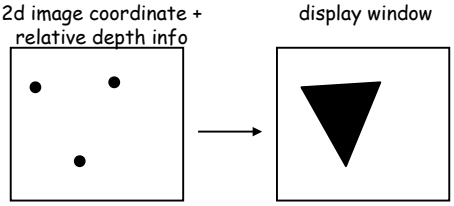
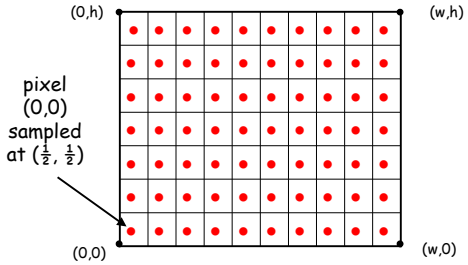
## scan conversion



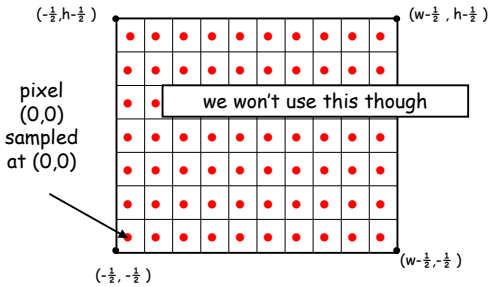



## display coordinates



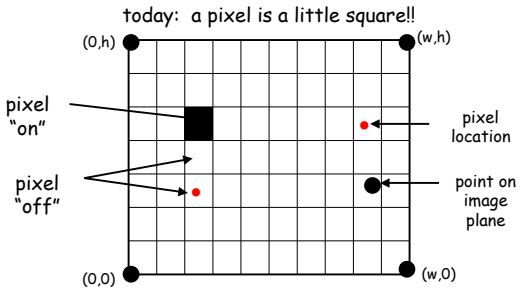



## display coordinates: alternative






## display coordinates



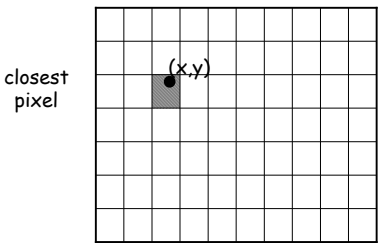



## scan conversion

- points
- line segments
- polygons

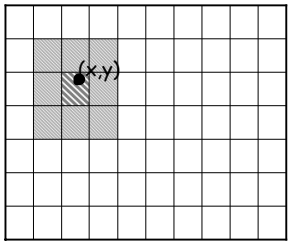


## scan conversion: point



## scan conversion: point

nearest neighbors





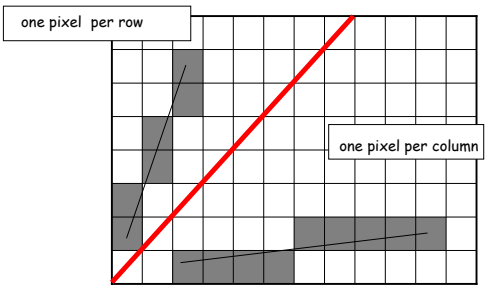

## line segments

**scan converting line segments**

- naïve algorithm
- midpoint algorithm
- bresenham's algorithm



## 1-pixel wide lines





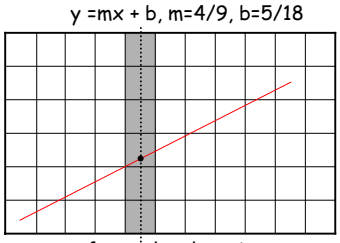

## scan conversion

- input: endpoint coordinates
- output: pixels to turn on (and their color) for a 1-pixel wide line segment



endpoints: ( $\frac{1}{2}$ , $\frac{1}{2}$ ) and ($9\frac{1}{2}$ ,$4\frac{1}{2}$ )

y =mx + b, m=4/9, b=5/18

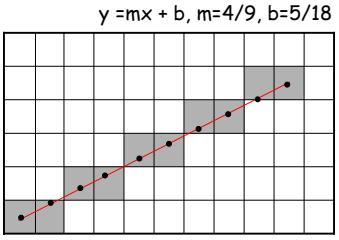


for each column i
compute the y-intercept at x=i+$\frac{1}{2}$



endpoints: ( $\frac{1}{2}$ , $\frac{1}{2}$ ) and ($9\frac{1}{2}$ ,$4\frac{1}{2}$ )

y =mx + b, m=4/9, b=5/18



for (i=0…9)
turn on pixel
(i, $\lfloor m*(i+\frac{1}{2})+b \rfloor$)

## naïve algorithm

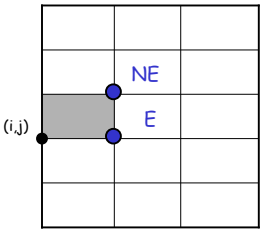
- input: endpoints $(x_0, y_0)$ to $(x_1, y_1)$
  for now we'll assume $x_0<x_1$!
- line: $y = mx + b$ where $m = (y_1 - y_0)/(x_1 - x_0)$, $b=y_0-mx_0$

for $i=\lfloor x_0 \rfloor \ldots \lfloor x_1 \rfloor$
  turn on pixel $(i, \lfloor m(i+\frac{1}{2}) + b \rfloor)$

is there a better/faster algorithm?
yes, we can avoid (almost all) multiplication!



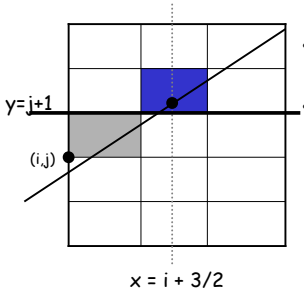
## midpoint algorithm: $0 \le m \le 1$




- suppose we've just turned on pixel (i,j)
- next we'll turn on
  NE: (i+1,j+1) or
  E: (i+1, j)



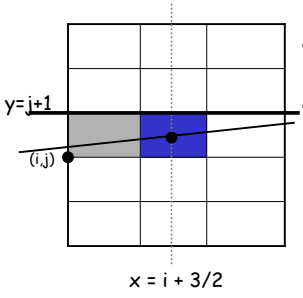
## midpoint algorithm: $0 \le m \le 1$




- suppose we've just turned on pixel (i,j)
- next we'll turn on
  - **NE: if the y intercept at x=i+3/2 is at least j+1**



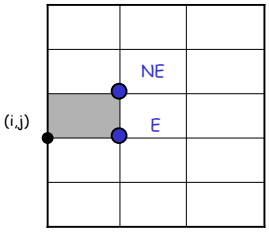
## midpoint algorithm: $0 \le m \le 1$




- suppose we've just turned on pixel (i,j)
- next we'll turn on
  - NE: if the y intercept at x=i+3/2 is at least j+1
  - **E: otherwise**



## midpoint algorithm: $0 \le m \le 1$



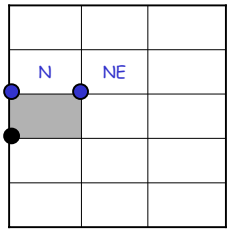

- suppose we've just turned on pixel (i,j)
- next we'll turn on
  - NE: if $j+1 \le m(i+3/2) + b$
  - E: otherwise



## midpoint algorithm: other cases

- similar rules

e.g. m>1



## advantage of midpoint algorithm

if the endpoints of the line segment have integer coordinates we can avoid floating point operations

this was a big deal in the dark ages!



## avoiding (almost all) multiplication (0≤m≤1)

- we just turned on pixel (i,j)
- next we'll turn on
  NE: if $j+1 \le m(i+3/2) + b$
  E: otherwise

$j+1 \le m(i+3/2) + b \quad \leftrightarrow \quad \Delta_x(j+1) \le \Delta_y (i+3/2) + \Delta_x b$

where $\Delta x = x_1 - x_0$ and $\Delta y = y_1 - y_0$

$\leftrightarrow \quad 2\Delta_x(j+1) \le \Delta_y (2i+3) + 2\Delta_x b$

$\leftrightarrow \quad 2\Delta_x(j+1) - \Delta_y (2i+3) - 2\Delta_x b \le 0$

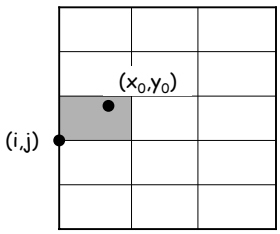
we can compute d incrementally without multiplication

$d$



## bresenham's algorithm (0≤m≤1)

1. turn on (i,j) where $i = \lfloor x_0 \rfloor$ $j = \lfloor y_0 \rfloor$




//find first pixel



## bresenham's algorithm (0≤m≤1)

2. $d = 2\Delta_x(j+1) - \Delta_y (2i+3) - 2\Delta_x b$

// initialize d



## bresenham's algorithm (0≤m≤1)

3. while $i \le x_1$ {
   if $d \le 0$ // go NE
   {
   }
   else // go E
   {
   }
}



## bresenham's algorithm (0≤m≤1)

3. while $i \le x_1$ {
   if $d \le 0$ // go NE
   {
   i++, j++
   turn on pixel (i,j)
   update d
   }
   else // go E
   {
   i++
   turn on pixel (i,j)
   update d
   }
}

## bresenham's algorithm ($0 \le m \le 1$)

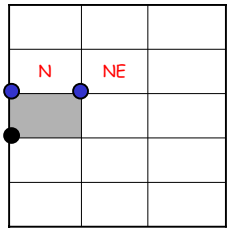
3. while $i \le x_1$ {
   if $d \le 0$ // go NE
   {
   i++, j++
   turn on pixel (i,j)
   $d = d + 2\Delta_x - 2\Delta_y$
   }
   else // go E
   {
   i++
   turn on pixel (i,j)
   $d = d - 2\Delta_y$
   }
}



## bresenham's algorithm: other cases

- similar rules

e.g. m>1



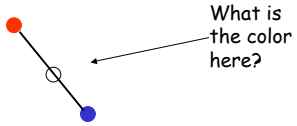



## scan conversion

- input: endpoint coordinates
- output: pixels to turn on for a 1-pixel wide line segment + **pixel color**



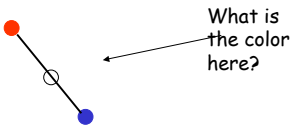
## shading models




Color is defined at vertices!!!!!!!



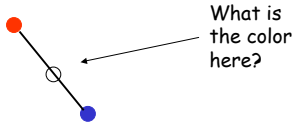
## flat shading




use color of first vertex

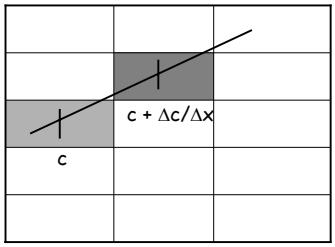


## smooth (gouraud) shading




interpolate

## interpolation computation



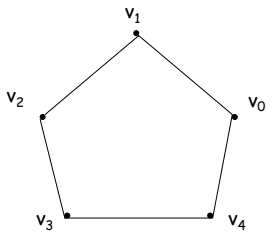



## scan conversion

- points
- line segments
- **polygons**



## polygon: $v_0,v_1,v_2,v_3,v_4$






## polygon: scan conversion

polygon($v_0$, …, $v_{n-1}$)
  for i=0 to n-1
    draw-line-segment($p_i$,$p_{i+1 \bmod n}$)



## scan conversion

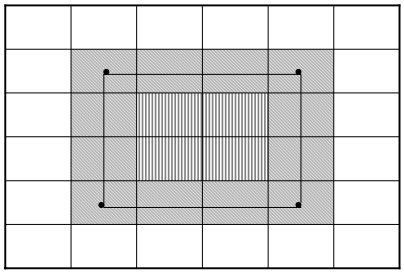
- points
- line segments
- polygons
  - filled



## scan conversion

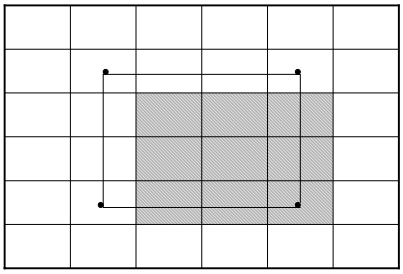
- input: vertex coordinates
- output: **pixels to turn on** (and their color) **for filled polygon**

## which pixels should be on?



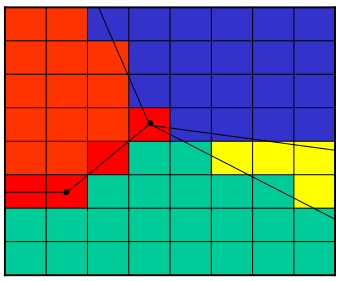


## here we get the same size!



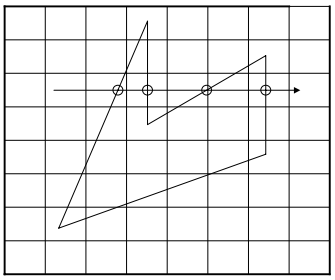


## tesselating polygons





## scan line algorithm

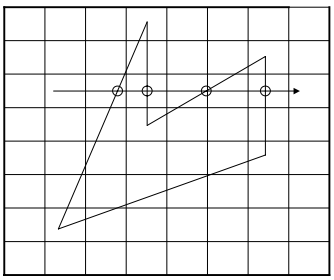


for each scan line

1. find edge/scan line intersection points
2. order by x-coordinate
3. use odd-even test to turn on pixels



## scan line algorithm

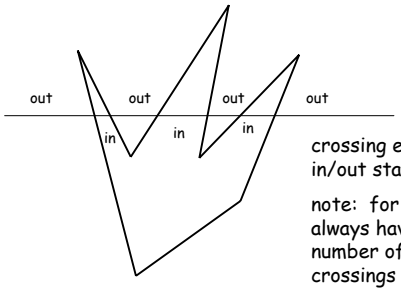


for each scan line

1. find edge/scan line intersection points
2. order by x-coordinate
3. **use odd-even test to turn on pixels**



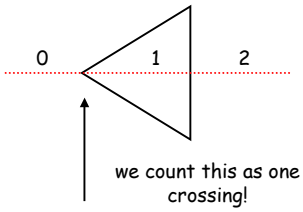
## odd-even test




crossing edge changes in/out state

note: for polygon we'll always have an even number of edge crossings

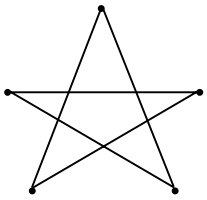
## odd-even test





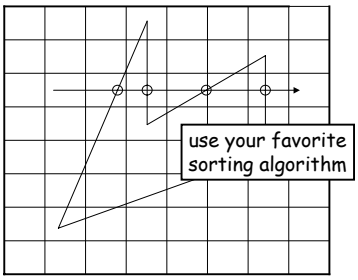

## odd-even test

buyer beware!





## scan line algorithm

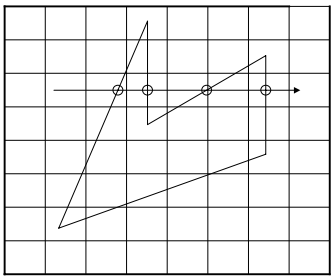



for each scan line

1. find edge/scan line intersection points
2. **order by x-coordinate**
3. use odd-even test to turn on pixels



## scan line algorithm

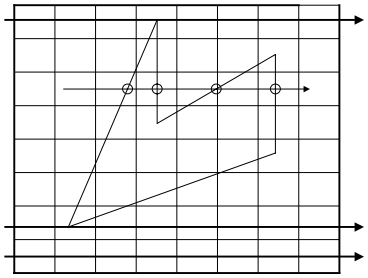


for each scan line

1. **find edge/scan line intersection points**
2. order by x-coordinate
3. use odd-even test to turn on pixels

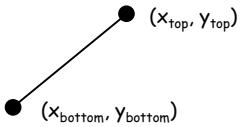


## scan line notation



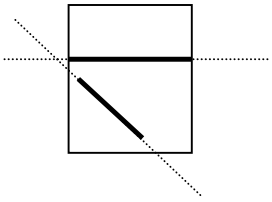



## edge notation




$y_{bottom} \leq y_{top}$

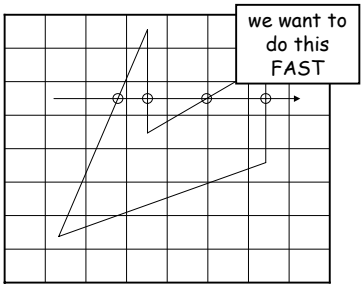
## naïve algorithm



for each edge of polygon
- compute intersection of current scan line & edge line
- check if intersection is on edge segment



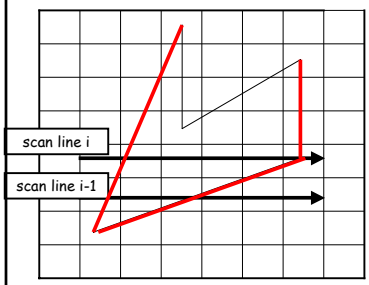
## scan line algorithm




for each scan line
1. **find edge/scan line intersection points**
2. order by x-coordinate
3. use odd-even test to turn on pixels



## exploit coherence




let $E_i$ be the edges that intersect scan line i

then $E_i$ =

$E_{i-1}$

+ edges with $i - \frac{1}{2} < y_{bottom} \leq i + \frac{1}{2}$

- edges with $y_{top} \leq i + \frac{1}{2}$



## data structures

- **edge table**
  - for each scan line i a list of edges with $i - \frac{1}{2} < y_{bottom} \leq i + \frac{1}{2}$
- active edge list
  - edges intersecting current scan line



## edge table (et)

| scan line | edge list |
|---|---|
| 5 | |
| 4 | |
| 3 | |
| 2 | |
| 1 | |
| 0 | |

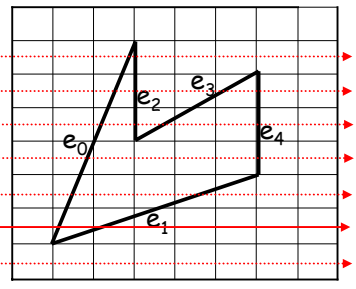
list of edges with with:

$2\frac{1}{2} < y_{bottom} \leq 3\frac{1}{2}$



## example edge table

| scan line | edge list |
|---|---|
| 7 | - |
| 6 | - |
| 5 | - |
| 4 | $e_2, e_3$ |
| 3 | $e_4$ |
| 2 | - |
| 1 | $e_0, e_1$ |
| 0 | - |

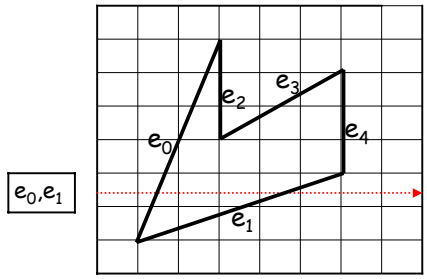


## data structures

- edge table
  - for each scan line *i* a list of edges with $i - \frac{1}{2} < y_{bottom} \leq i + \frac{1}{2}$
- **active edge list**
  - edges intersecting current scan line

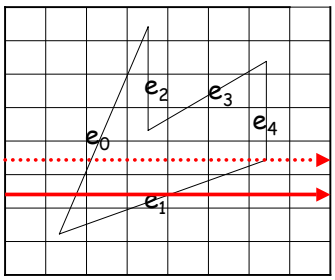


## example active edge list





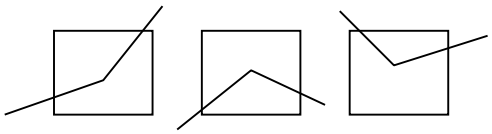

## ael update




ael = ael + et[i] – edges with $y_{top} \leq i+\frac{1}{2}$



## ael changes

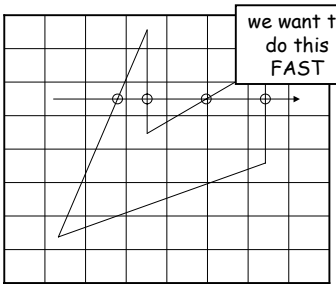


end/begin two end two begin

ael always contains an even number of edges!



## scan line algorithm



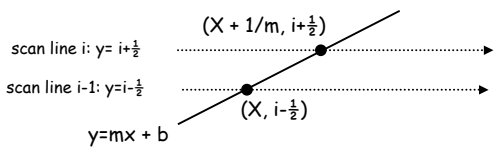
we want to do this FAST

for each scan line

1. **find edge/scan line intersection points**
2. order by x-coordinate
3. use odd-even test to turn on pixels



## intersection point calculation




all we need to store is x-intercept, X, and inverse slope

## edge record at scan line i

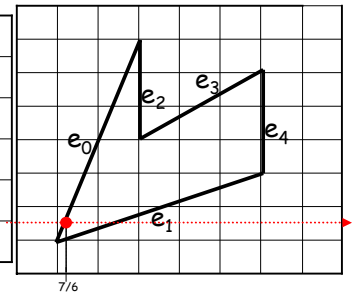
- $y_{bottom}$
- $y_{top}$
- 1/m
- $x_{int}$ : x-intercept at scan line i

initialize to x-intercept at first scan line the edge intersects



## initialize edge records

| edge | $y_{bottom}$ | $y_{top}$ | $x_{int}$ | 1/m |
|---|---|---|---|---|
| $e_0$ | 1 | 7 | 7/6 | 1/3 |
| $e_1$ | 1 | 3 | 9/4 | 5/2 |
| $e_2$ | 4 | 7 | 3 | 0 |
| $e_3$ | 4 | 6 | 15/4 | 3/2 |
| $e_4$ | 3 | 6 | 6 | 0 |






## scan line algorithm

```
build et
scanLine=-1
ael=φ
while scanLine < h
    scanLine ++
    for each edge in ael:  x_int += 1/m
    ael += et[scanLine]
    ael -= {edges with y_top ≤ scanLine + ½ }
    sort edges in ael by x_int
    compute "on" pixels by odd-even rule
```



## scan line algorithm

```
build et
scanLine=-1
ael=φ
while scanLine < h
    scanLine ++
    for each edge in ael:  x_int += 1/m
    ael += et[scanLine]
    ael -= {edges with y_top ≤ scanLine + ½ }
    sort edges in ael by x_int
```



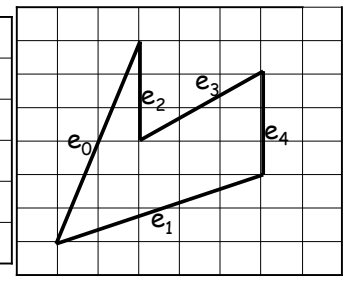
## scan line algorithm

```
//compute "on" pixels by odd-even rule
for k=0 … aelNumEdges/2
    for j + ½  > aelEdges[2k].x_int  and
        j  + ½ ≤ aelEdges[2k+1].x_int
                turn on pixels (j, scanLine)
```



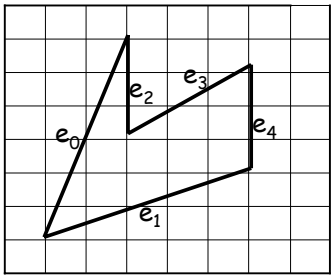
## initialize edge records

| edge | $y_{bottom}$ | $y_{top}$ | $x_{int}$ | 1/m |
|---|---|---|---|---|
| $e_0$ | 1 | 7 | 7/6 | 1/3 |
| $e_1$ | 1 | 3 | 9/4 | 5/2 |
| $e_2$ | 4 | 7 | 3 | 0 |
| $e_3$ | 4 | 6 | 15/4 | 3/2 |
| $e_4$ | 3 | 6 | 6 | 0 |



## initialize edge table

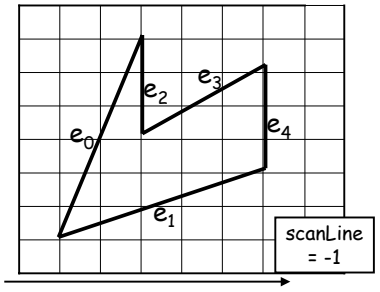
| scan line | Edges |
|---|---|
| 6 | - |
| 5 | - |
| 4 | $e_2$,$e_3$ |
| 3 | $e_4$ |
| 2 | - |
| 1 | $e_0$,$e_1$ |
| 0 | - |






## example

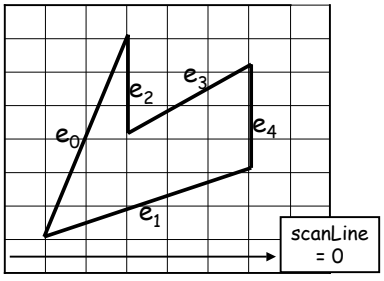
ael = $\phi$






## example

ael = $\phi$

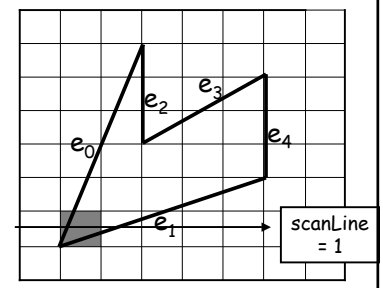





## example

ael sorted by $x_{int}$

| edge | $x_{int}$ | ... |
|---|---|---|
| $e_0$ | 7/6 | ... |
| $e_1$ | 9/4 | ... |

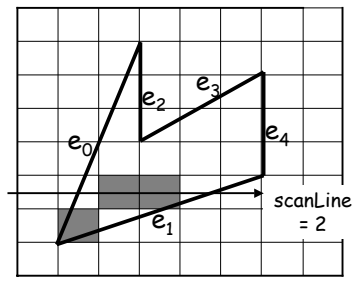





## example

ael sorted by $x_{int}$

| edge | $x_{int}$ | ... |
|---|---|---|
| $e_0$ | 9/6 | ... |
| $e_1$ | 19/4 | ... |

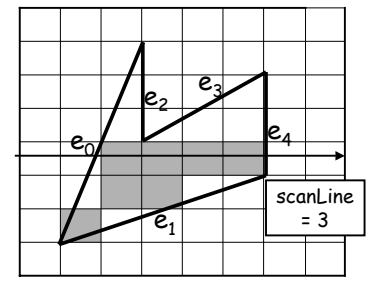





## example

ael sorted by $x_{int}$

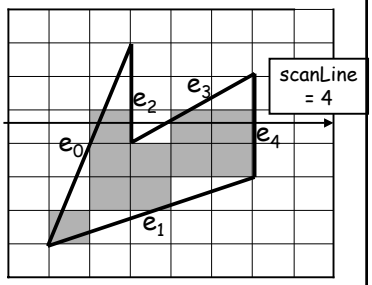
| edge | $x_{int}$ | ... |
|---|---|---|
| $e_0$ | 11/6 | ... |
| $e_4$ | 6 | ... |



## example

ael sorted by $x_{int}$

| edge | $x_{int}$ | ... |
|---|---|---|
| $e_0$ | 13/6 | ... |
| $e_2$ | 3 | ... |
| $e_3$ | 15/4 | ... |
| $e_4$ | 6 | ... |

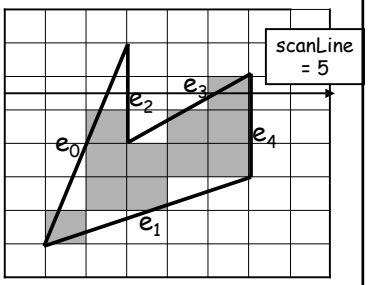





## example

ael sorted by $x_{int}$

| edge | $x_{int}$ | ... |
|---|---|---|
| $e_0$ | 15/6 | ... |
| $e_2$ | 3 | ... |
| $e_3$ | 21/4 | ... |
| $e_4$ | 6 | ... |

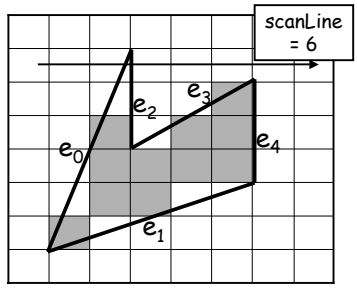





## example

ael sorted by $x_{int}$

| edge | $x_{int}$ | ... |
|---|---|---|
| $e_0$ | 17/6 | ... |
| $e_2$ | 3 | ... |

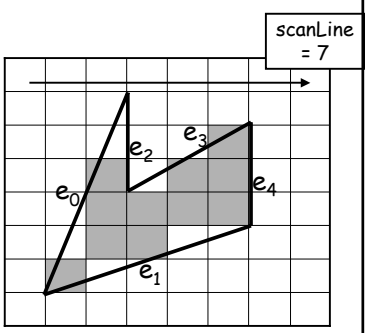





## example

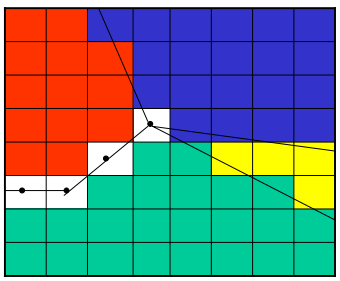
ael sorted by $x_{int}$

| edge | $x_{int}$ | ... |
|---|---|---|



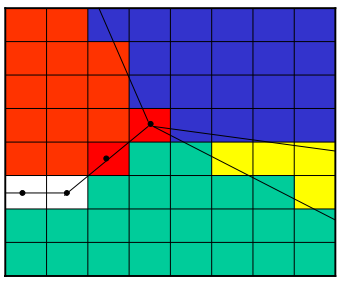



## tesselation: center claims

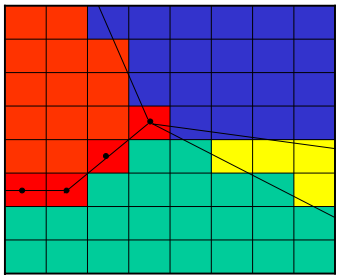




## tie breaker 1: left owns



## tie breaker 2: above owns





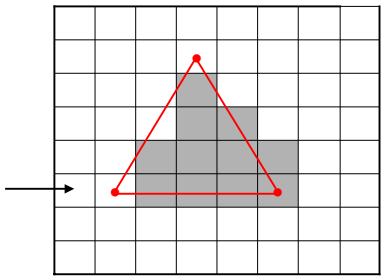
## exercise

- where in the algorithm are these tie-breaking rules specified?



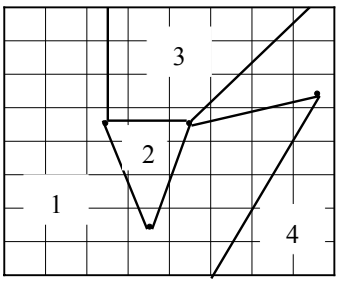
## horizontal edges

what is in aet when scanLine = 2





## Exercise



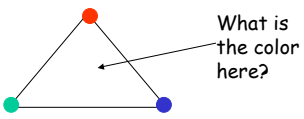



## scan conversion

- input: vertex coordinates
- output: pixels to turn on for filled polygon + **pixel color**

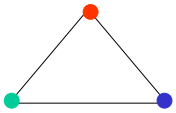


## shading models




Color is defined at vertices!!!!!!!

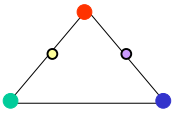
## flat shading



Color entire polygon the color of first vertex.



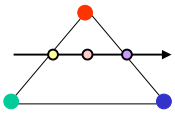
## smooth shading



1. Interpolate along edges.



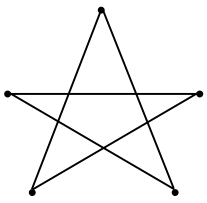
## smooth shading



1. Interpolate along edges.
2. Interpolate along scan line between edges.



## what happens here?





## edge record at scan line i

- $y_{bottom}$
- $y_{top}$
- $1/m$
- $x_{int}$
- $c_{int}$ ← color on edge at $(x_{int}, i)$
- $\Delta_c/\Delta_x$ ← color increment

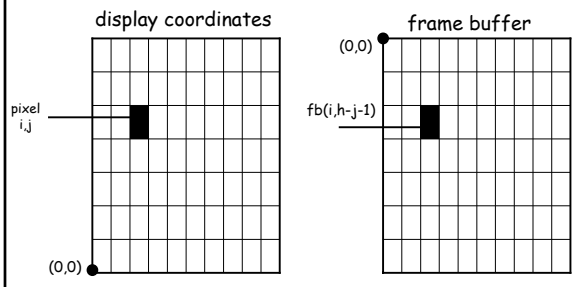


## scan conversion

- points
- lines segments
- polygons
  - filled

the frame and z buffers and hidden surface removal

## display coordinates vs. frame buffer





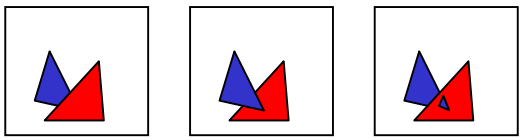

## scan conversion

- points
- lines segments
- polygons
  - filled

the frame & z buffers and hidden surface removal



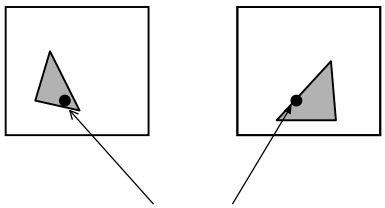
## hidden surface removal



which is right?



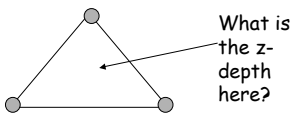
## hidden surface removal



compare z-depth of corresponding points on 3d surfaces



## z-depth

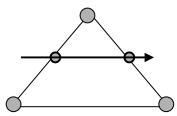



z-depth is defined at vertices!!!!!!!

so interpolate



## z-depth



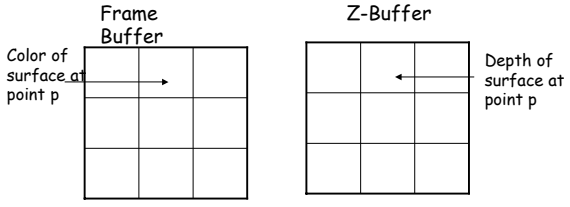
1. Interpolate along edges.
2. Interpolate along scan line.

## edge record at scan line i

- $y_{top}$
- $x_i$
- 1/m
- $c_i$
- $\Delta_i$
- $z_i$ ← depth on edge at $(i,x_i)$
- $\delta_i$ ← depth increment: to compute $z_i$ incrementally



## z-buffering






## initialize the z-buffer

| | | |
|---|---|---|
| -1 | -1 | -1 |
| -1 | -1 | -1 |
| -1 | -1 | -1 |



## scan conversion

- Without z-buffering:
  fb(i,h-j)=currcolor:
- With z-buffering

```
// z val is the normalized depth of the point on the polygon
   //that projects to point (i,j)
Compute zval
If zval > zb(i,h-j) {
        fb(i,h-j) = currcolor
        zb(i,h-j) = zval
        }
```